Lecture 1.

Units and Dimensions
In mathematics,

If $x = 500$ and $y = 100$, then $(x + y) = 600$

In engineering,

If $x = 500\text{m}$ and $y = 100\text{m}$, then $(x + y) = 600\text{m}$

But,

If $x = 500\text{m}$ and $y = 100\text{kg}$, then $(x + y) = 600\text{??}$
Why Do We Need Units?

Units are important for effective communication and standardization of measurements.

Image Source: http://lamar.colostate.edu/~hillger/feet-to-meters_1.jpg
The “Gimli Glider” Incident (23 July 1983)

Pounds vs Kilograms

Image Source: http://upload.wikimedia.org
The Mars Climate Orbiter Incident (23 September 1999)

Newton vs Pound Force

### The 7 Fundamental (Base) Dimensions

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass</td>
<td>m</td>
</tr>
<tr>
<td>Length</td>
<td>L</td>
</tr>
<tr>
<td>Time</td>
<td>θ</td>
</tr>
<tr>
<td>Temperature</td>
<td>T</td>
</tr>
<tr>
<td>Mole</td>
<td>n</td>
</tr>
<tr>
<td>Luminosity</td>
<td>c</td>
</tr>
<tr>
<td>Electric Current</td>
<td>I</td>
</tr>
<tr>
<td>Dimension</td>
<td>SI Unit</td>
</tr>
<tr>
<td>-------------</td>
<td>------------------</td>
</tr>
<tr>
<td>Mass</td>
<td>kilogram (kg)</td>
</tr>
<tr>
<td>Length</td>
<td>meter (m)</td>
</tr>
<tr>
<td>Time</td>
<td>second (s)</td>
</tr>
<tr>
<td>Temperature</td>
<td>Kelvin (K)</td>
</tr>
<tr>
<td>Mole</td>
<td>gram mole (gmol)</td>
</tr>
</tbody>
</table>
## Secondary (Derived) Dimensions

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>Area</td>
<td>$L^2$</td>
</tr>
<tr>
<td>Volume</td>
<td>$L^3$</td>
</tr>
<tr>
<td>Velocity</td>
<td>$L/\theta$</td>
</tr>
<tr>
<td>Acceleration</td>
<td>$L/\theta^2$</td>
</tr>
<tr>
<td>Force</td>
<td>$m \cdot (L/\theta^2)$</td>
</tr>
<tr>
<td>Pressure</td>
<td>$m \cdot (L/\theta^2)/L^2 = m/\theta^2 \cdot L$</td>
</tr>
<tr>
<td>Energy</td>
<td>$m \cdot (L/\theta^2) \cdot L = m \cdot (L^2/\theta^2)$</td>
</tr>
<tr>
<td>Dimension</td>
<td>SI Unit</td>
</tr>
<tr>
<td>---------------</td>
<td>---------------</td>
</tr>
<tr>
<td>Volume</td>
<td>m$^3$</td>
</tr>
<tr>
<td>Acceleration</td>
<td>m/s</td>
</tr>
<tr>
<td>Force</td>
<td>kg $\cdot$ m/s$^2$</td>
</tr>
<tr>
<td>Pressure</td>
<td>kg / (m $\cdot$ s$^2$)</td>
</tr>
<tr>
<td>Energy</td>
<td>kg $\cdot$ (m$^2$/s$^2$)</td>
</tr>
</tbody>
</table>
## Defined Equivalent Units

<table>
<thead>
<tr>
<th>Dimension</th>
<th>SI Unit</th>
<th>Am. Eng. Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Force</strong></td>
<td>$1 \text{kg} \cdot \text{m/s}^2 = 1 \text{N}$</td>
<td>$32.174 \text{lb}_m \cdot \text{ft/s}^2 = 1 \text{lb}_f$</td>
</tr>
<tr>
<td><strong>Pressure</strong></td>
<td>$1 \text{ kg} / (\text{m} \cdot \text{s}^2) = 1 \text{ N/m}^2 = 1 \text{ Pa}$</td>
<td>$32.174 \text{ lb}_m / (\text{ft} \cdot \text{s}^2) = 1 \text{ lbf/ft}^2 = (1/144) \text{ lbf/in}^2 (\text{psi})$</td>
</tr>
<tr>
<td><strong>Energy</strong></td>
<td>$1 \text{ kg} \cdot (\text{m}^2/\text{s}^2) = 1 \text{ N} \cdot \text{m} = 1 \text{ J}$</td>
<td>$32.174 \text{ lb}_m \cdot (\text{ft}^2/\text{s}^2) = 1 \text{ ft-lb}$</td>
</tr>
</tbody>
</table>
### Conversion of Units: Single Measurements

The equivalence between two units of the same measurement may be defined in terms of a ratio (conversion factor):

\[
\frac{\text{Old Unit}}{\text{New Unit}} = \frac{\text{New Unit}}{\text{Old Unit}}
\]

\[
\frac{1}{\text{Old Unit}} \left( \frac{\text{Old Unit}}{\text{New Unit}} \right) = \frac{1}{\text{New Unit}}
\]
Conversion of Units: Single Measurements

\[ 500 \text{ kg} \left( \frac{2.2 \text{ lbm}}{\text{kg}} \right) = 1100 \text{ lbm} \]

\[ \frac{300}{\text{cm}^2} \left( \frac{1 \text{ cm}}{10 \text{ mm}} \right)^2 = \frac{300}{\text{cm}^2} \left( \frac{1 \text{ cm}^2}{10^2 \text{ mm}^2} \right) = \frac{3}{\text{mm}^2} \]

\[ \frac{1}{\text{s}^2} \left( \frac{3600 \text{ s}}{1 \text{ h}} \right)^2 \left( \frac{24 \text{ h}}{1 \text{ d}} \right)^2 \left( \frac{365 \text{ d}}{1 \text{ yr}} \right)^2 \left( \frac{1 \text{ m}}{100 \text{ cm}} \right) \left( \frac{1 \text{ km}}{1000 \text{ m}} \right) = 9.95 \times 10^9 \frac{\text{km}}{\text{yr}^2} \]
Consider the following equation of motion:

\[ D \text{ (ft)} = 3 \, t\text{(s)} - 4 \]

Derive an equivalent equation for distance in meters and time in minutes.

**Step 1.** Define new variables \( D'(m) \) and \( t'(\text{min}) \).

**Step 2.** Define the old variables in terms of the new variable.
Conversion of Units: Equations or Formula

\[
D(\text{ft}) = D'(\text{m}) \times \left( \frac{3.2808 \text{ ft}}{1 \text{ m}} \right) \text{ or } D = 3.2808D'
\]

\[
t(\text{s}) = t'(\text{min}) \times \left( \frac{60 \text{ s}}{1 \text{ min}} \right) \text{ or } t = 60t'
\]

Step 3. Substitute these equivalence relations into the original equation.

\[
(3.2808D') = 3 \times (60t') - 4
\]

Simplifying,

\[
D'(\text{m}) = 55t'(\text{min}) - 1.22
\]
The numerical value of two or more quantities can be added/subtracted only if the units of the quantities are the same.

5 kilograms + 3 meters = no physical meaning

10 feet + 3 meters = has physical meaning

10 feet + 9.84 feet = 19.94 feet
Multiplication and division can be done on quantities with unlike units but the units can only be cancelled or merged if they are identical.

5 kilograms x 3 meters = 15 kg-m

3 m²/60 cm = 0.05 m²/cm

3 m²/0.6 m = 5 m²/m = 5 m
Reynolds Number Calculation

Reynolds number is calculated as:

$$\text{Reynolds Number} = \frac{\rho D v}{\mu}$$

where

- $\rho$ = density of the fluid (kg/m$^3$)
- $D$ = diameter of pipe (m)
- $v$ = mean velocity of fluid (m/s)
- $\mu$ = dynamic viscosity (kg/m $\cdot$ s)

What is the net dimension of Reynolds Number?
Importance of Dimensionless Quantities

Used in arguments of special functions such as exponential, logarithmic, or trigonometric functions.

\[ e^{20} \text{ is possible but } e^{(20 \text{ft})} \text{ is undefined} \]

\[ \cos(20) \text{ is possible but } \cos(20 \text{ ft}) \text{ is undefined} \]
Importance of Dimensionless Quantities

Consider the Arrhenius Equation:

\[ k = A e^{\frac{-E_a}{RT}} \]

If \( E_a \) is activation in cal/mol and \( T \) is temperature in K, what is the unit of \( R \)?

To make the argument of the exponential function dimensionless, \( R \) must have a unit of \((\text{cal/mol-K})\).
Every valid equation must be dimensionally consistent.

Each term in the equation must have the same net dimensions and units as every other term to which it is added, subtracted, or equated.

\[ A + B = C - DE \]

If \( A \) has a dimension of \( L^3 \), then

1. \( B \) must have a dimension of \( L^3 \) since it is added to \( A \).
2. \( (A + B) \) has a net dimension of \( L^3 \).
3. \( (C - DE) \) must have a net dimension of \( L^3 \).
4. \( C \) and \( DE \) have a dimension of \( L^3 \).
Example on Dimensional Consistency

The density of a fluid is given by the empirical equation

\[ \rho = 70.5 \exp(8.27 \times 10^{-7} P) \]

where \( \rho \) = density in \((\text{lbm/ft}^3)\) and \( P \) = pressure \((\text{lbf/in}^2)\).

a. What are the units of 70.5 and 8.27x10^{-7}?

b. Derive a formula for \( \rho \) \((\text{g/cm}^3)\) and \( P \) \((\text{N/m}^2)\)
\rho = 70.5 \exp(8.27 \times 10^{-7} \ P)

1. Since the exponential part is dimensionless, then 70.5 must have the same unit as \rho which is (lbm/ft^3).

2. Since the argument of the exponential function must be dimensionless, then 8.27 \times 10^{-7} must have a unit of (in.^2/lbf) which is a reciprocal to the unit of P.
1. Define new variables $\rho'$ (g/cm$^3$) and $P'$ (N/m$^2$).

2. Express the old variables in terms of the new variables.

\[ \rho\left(\frac{\text{lbm}}{\text{ft}^3}\right) = \rho'\left(\frac{\text{g}}{\text{cm}^3}\right)\left(\frac{1 \text{ lbm}}{453.593 \text{ g}}\right)\left(\frac{28,317 \text{ cm}^3}{1 \text{ ft}^3}\right) = 62.43\rho' \]

\[ P\left(\frac{\text{lbf}}{\text{in}^2}\right) = P'\left(\frac{\text{N}}{\text{m}^2}\right)\left(\frac{0.2248 \text{ lbf}}{1 \text{ N}}\right)\left(\frac{1 \text{ m}^2}{39.37 \text{ in}^2}\right) = 1.45 \times 10^{-4}P' \]
3. Substitute the equivalence relations into the original equation.

\[ \rho = 70.5 \exp\left(8.27 \times 10^{-7}P\right) \]

\[ 62.43\rho' = 70.5 \exp\left[8.27 \times 10^{-7} \left(1.45 \times 10^{-4}P'\right)\right] \]

Simplifying,

\[ \rho' \left(\frac{g}{cm^3}\right) = 1.13 \exp\left[1.20 \times 10^{-10}P'\left(\frac{N}{m^2}\right)\right] \]