Lecture 1.

Units and Dimensions

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Mathematics and Engineering

In mathematics,

If x = 500 and y = 100, then (x + y) = 600

In engineering,

If x = 500m and y = 100m, then (x + y) = 600m

But,

If x = 500m and y = 100kg, then (x + y) = 600???

Why Do We Need Units?

Units are important for effective communication and standardization of measurements

Image Source: http://lamar.colostate.edu/~hillger/feet-to-meters_1.jpg

The "Gimli Glider" Incident (23 July 1983)

Pounds vs Kilograms

Image Source: http://upload.wikimedia.org

The Mars Climate Orbiter Incident (23 September 1999)

Newton vs Pound Force

Image Source: http://www.jpl.nasa.gov/... /climate-orbiter-browse.jpg

The 7 Fundamental (Base) Dimensions

Dimension	Symbol
Mass	m
Length	L
Time	θ
Temperature	T
Mole	n
Luminosity	c
Electric Current	I

SI and American Engineering System Units for Fundamental Dimensions

Dimension	SI Unit	Am. Eng. Unit
Mass	kilogram **(kg)**	poundmass **(lb_m)**
Length	meter **(m)**	foot **(ft)**
Time	second **(s)**	second **(s)**
Temperature	Kelvin **(K)**	Rankine **(^0R)**
Mole	gram mole **(gmol)**	pound mole **(lbmol)**

Secondary (Derived) Dimensions

Dimension	Symbol
Area	L^2
Volume	L^3
Velocity	L/θ
Acceleration	L/θ^2
Force	$m \cdot (L/\theta^2)$
Pressure	$m \cdot (L/\theta^2)/L^2 = m/\theta^2 \cdot L$
Energy	$m \cdot (L/\theta^2) \cdot L = m \cdot (L^2/\theta^2)$

SI and American Engineering System Units for Secondary Dimensions

Dimension	SI Unit	Am. Eng. Unit
Volume	m^3	ft^3
Acceleration	m/s	ft/s
Force	$kg \cdot m/s^2$	$lb_m \cdot ft/s^2$
Pressure	$kg/(m \cdot s^2)$	$lb_m/(ft \cdot s^2)$
Energy	$kg \cdot (m^2/s^2)$	$lb_m \cdot (ft^2/s^2)$

Defined Equivalent Units

Dimension	SI Unit	Am. Eng. Unit
Force	$1 kg \cdot m/s^2$ **= 1 N**	$32.174\ lb_m \cdot ft/s^2$ **= 1 lb_f**
Pressure	$1\ kg/(m \cdot s^2)$ **= 1 N/m^2 = 1 Pa**	$32.174\ lb_m/(ft \cdot s^2)$ **= 1 lbf/ft^2 = (1/144) lbf/in^2 (psi)**
Energy	$1\ kg \cdot (m^2/s^2)$ **= 1 N · m = 1 J**	$32.174\ lb_m \cdot (ft^2/s^2)$ **= 1 ft · lb_f**

Conversion of Units: Single Measurements

The equivalence between two units of the same measurement may be defined in terms of a ratio (conversion factor):

$$\textbf{Old Unit}\left(\frac{\textbf{New Unit}}{\textbf{Old Unit}}\right)=\textbf{New Unit}$$

$$\frac{\textbf{1}}{\textbf{Old Unit}}\left(\frac{\textbf{Old Unit}}{\textbf{New Unit}}\right)=\frac{\textbf{1}}{\textbf{New Unit}}$$

Conversion of Units: Single Measurements

$$500\ \text{kg}\left(\frac{2.2\ \text{lbm}}{\text{kg}}\right)=1100\ \text{lbm}$$

$$\frac{300}{\text{cm}^2}\left(\frac{1\ \text{cm}}{10\ \text{mm}}\right)^2=\frac{300}{\text{cm}^2}\left(\frac{1\ \text{cm}^2}{10^2\ \text{mm}^2}\right)=\frac{3}{\text{mm}^2}$$

$$1\frac{\text{cm}}{\text{s}^2}\left(\frac{3600\,\text{s}}{1\,\text{h}}\right)^2\left(\frac{24\,\text{h}}{1\,\text{d}}\right)^2\left(\frac{365\,\text{d}}{1\,\text{yr}}\right)^2\left(\frac{1\,\text{m}}{100\,\text{cm}}\right)\left(\frac{1\,\text{km}}{1000\,\text{m}}\right)=9.95\,\text{x}\,10^9\,\frac{\text{km}}{\text{yr}^2}$$

Conversion of Units: Equations or Formula

Consider the following equation of motion:

$$\mathbf{D\ (ft) = 3\ t(s) - 4}$$

Derive an equivalent equation for distance in meters and time in minutes.

Step 1. Define new variables D'(m) and t'(min).

Step 2. Define the old variables in terms of the new variable.

Conversion of Units: Equations or Formula

$$D(ft) = D'(m) \times \left(\frac{3.2808\ ft}{1\ m}\right) \text{ or } D = 3.2808D'$$

$$t(s) = t'(min) \times \left(\frac{60\ s}{1\ min}\right) \text{ or } t = 60t'$$

Step 3. Substitute these equivalence relations into the original equation.

$$(3.2808D') = 3\ (60t') - 4$$

Simplifying,

$$D'\ (m) = 55t'(min) - 1.22$$

Operation on Units: Addition and Subtraction

The numerical value of two or more quantities can be added/subtracted only if the units of the quantities are the same.

5 kilograms + 3 meters = no physical meaning

10 feet + 3 meters = has physical meaning

10 feet + 9.84 feet = 19.94 feet

Operation on Units: Multiplication and Division

Multiplication and division can be done on quantities with unlike units but the units can only be cancelled or merged if they are identical.

5 kilograms x 3 meters = 15 kg-m

3 m^2/60 cm = 0.05 m^2/cm

3 m2/0.6 m = 5 m^2/m = 5 m

Dimensionless Quantities

Reynolds Number Calculation

Reynolds number is calculated as:

$$\textbf{Reynolds Number} = \frac{\rho D v}{\mu}$$

where

ρ	=	density of the fluid (kg/m^3)
D	=	diameter of pipe (m)
v	=	mean velocity of fluid (m/s)
μ	=	dynamic viscosity ($kg/m \cdot s$)

What is the net dimension of Reynolds Number?

Dimensionless Quantities

Importance of Dimensionless Quantities

Used in arguments of special functions such as exponential, logarithmic, or trigonometric functions.

e^{20} is possible but $e^{(20ft)}$ is undefined

cos(20) is possible but cos(20 ft) is undefined

Dimensionless Quantities

Importance of Dimensionless Quantities

Consider the Arrhenius Equation:

$$\mathbf{k = Ae^{-\frac{E_a}{RT}}}$$

If E_a is activation in cal/mol and T is temperature in K, what is the unit of R?

To make the argument of the exponential function dimensionless, R must have a unit of <u>(cal/mol-K)</u>.

Dimensional Consistency

Every valid equation must be dimensionally consistent.

Each term in the equation must have the same net dimensions and units as every other term to which it is added, subtracted, or equated.

$$\mathbf{A + B = C - DE}$$

If A has a dimension of L^3, then

1. B must have a dimension of L^3 since it is added to A.
2. (A + B) has a net dimension of L^3.
3. (C – DE) must have a net dimension of L^3
4. C and DE have a dimension of L^3.

Dimensional Consistency

Example on Dimensional Consistency

The density of a fluid is given by the empirical equation

$$\rho = 70.5 \exp(8.27 \times 10^{-7} P)$$

where ρ = density in (lbm/ft^3) and P = pressure (lbf/in^2).

a. What are the units of 70.5 and 8.27×10^{-7}?

b. Derive a formula for ρ (g/cm^3) and P (N/m^2)

Dimensional Consistency

$$\rho = 70.5\ \underline{\exp(8.27 \times 10^{-7}\ P)}$$

1. Since the exponential part is dimensionless, then 70.5 must have the same unit as ρ which is $\underline{(lbm/ft^3)}$.

2. Since the argument of the exponential function must be dimensionless, then 8.27 x 10-7 must have a unit of $\underline{(in.^2/lbf)}$ which is a reciprocal to the unit of P.

Dimensional Consistency

1. Define new variables ρ' (g/cm^3) and P' (N/m^2).

2. Express the old variables in terms of the new variables.

$$\rho\left(\frac{\text{lbm}}{\text{ft}^3}\right) = \rho'\left(\frac{\text{g}}{\text{cm}^3}\right)\left(\frac{1\ \text{lbm}}{453.593\ \text{g}}\right)\left(\frac{28{,}317\ \text{cm}^3}{1\ \text{ft}^3}\right) = 62.43\rho'$$

$$P\left(\frac{\text{lbf}}{\text{in}^2}\right) = P'\left(\frac{\text{N}}{\text{m}^2}\right)\left(\frac{0.2248\ \text{lbf}}{1\ \text{N}}\right)\left(\frac{1\ \text{m}^2}{39.37\ \text{in}^2}\right) = 1.45\text{x}10^{-4}P'$$

Dimensional Consistency

3. Substitute the equivalence relations into the original equation.

$$\rho = 70.5 \exp\left(8.27 \text{ x } 10^{-7} P\right)$$

$$62.43\rho' = 70.5 \exp\left[8.27 \text{ x } 10^{-7}\left(1.45 \text{ x } 10^{-4} P'\right)\right]$$

Simplifying,

$$\rho'\left(\frac{g}{cm^3}\right) = 1.13 \exp\left[1.20 \text{ x } 10^{-10} P'\left(\frac{N}{m^2}\right)\right]$$