15

Energy Balances on Non-Reactive Process

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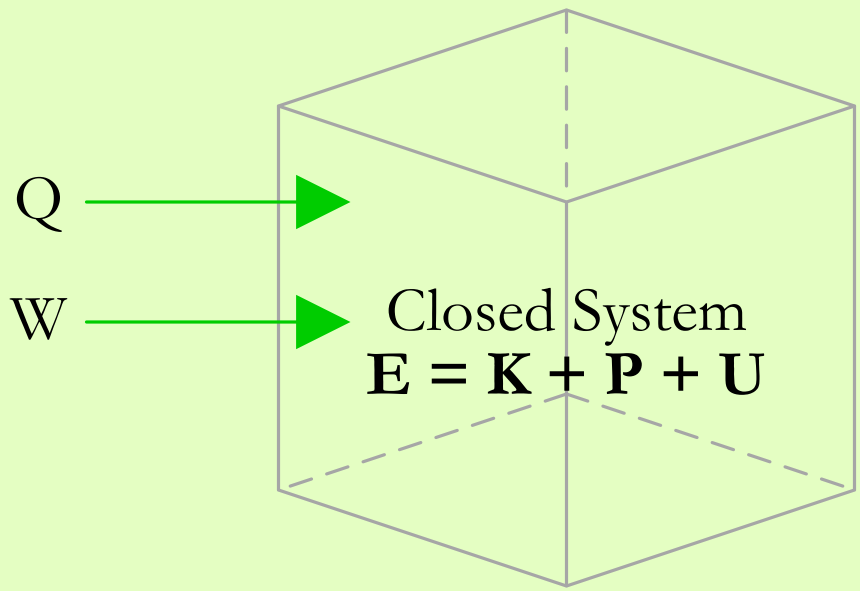
15.1 Energy Balance on a Closed System



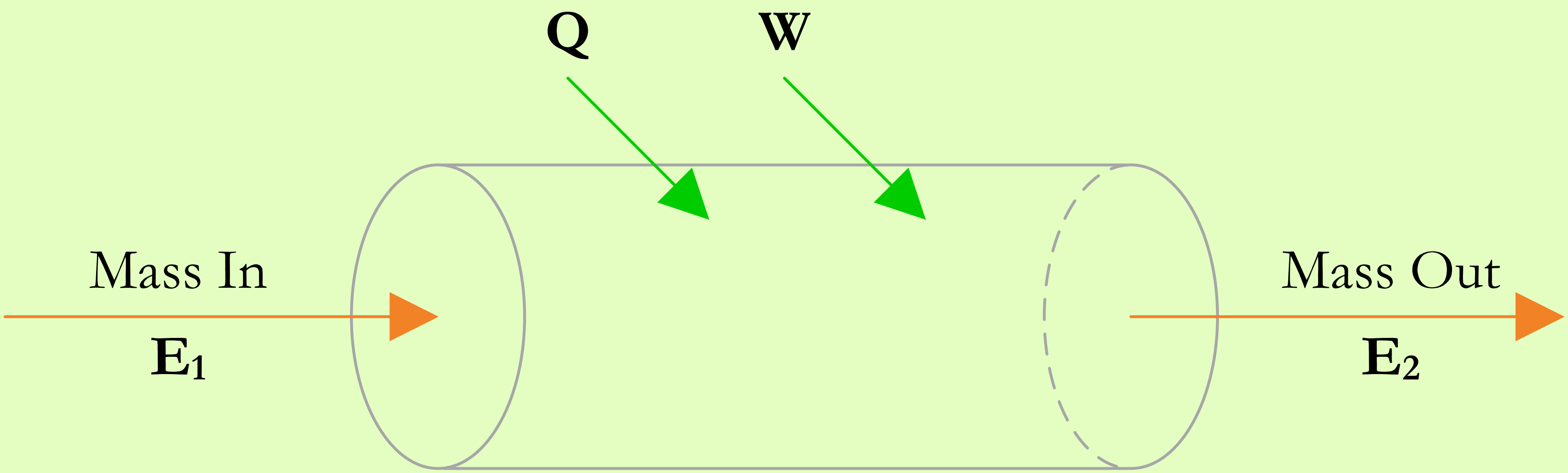
$$\Delta E = Q - W$$

$$\Delta K + \Delta P + \Delta U = Q - W$$

$$\Delta(mv^2/2) + \Delta(mgh) + \Delta U = Q - W$$

$$m/2\Delta v^2 + mg\Delta h + \Delta U = Q - W$$

15.2 Energy Balance on an Open System



$$\Delta E = Q - W$$

$$E_2 - E_1 = Q - W$$

If there are multiple inlets and outlets,

$$\Sigma E_2 - \Sigma E_1 = Q - W$$

The work appearing in the equation is the combined flow work and shaft work:

$$\mathbf{W = W_F + W_S}$$

W_F = flow work; work that is necessary to get mass into and out of the system

W_S = shaft work; work produced or required beside getting mass into and out of the system.

Hence,

$$\mathbf{\Sigma E_2 - \Sigma E_1 = Q - (W_F + W_S)}$$

The net flow work is determined as

$$\mathbf{W_F = (W_F)_2 - (W_F)_1}$$

The flow work is usually expressed in terms of pressure and volume:

$$\mathbf{W_F = (PV)_2 - (PV)_1}$$

For multiple inlets and outlets,

$$\mathbf{W_F = \Sigma(PV)_2 - \Sigma(PV)_1}$$

The energy balance becomes

$$\Sigma E_2 - \Sigma E_1 = Q - [(\Sigma(PV)_2 - \Sigma(PV)_1) + W_S]$$

Since E = K + P + U, then

$$\Sigma(K + P + U)_2 - \Sigma(K + P + U)_1 = Q - [(\Sigma(PV)_2 - \Sigma(PV)_1) + W_S]$$

Rearranging the terms,

$$\Sigma(K + P + U + PV)_2 - \Sigma(K + P + U + PV)_1 = Q - W_S$$
$$\Sigma(K + P + H)_2 - \Sigma(K + P + H)_1 = Q - W_S$$

Example 15-1. Compression of an Ideal Gas in a Cylinder

A cylinder with a movable piston contains 4.00 liters of a gas at 30^0C and 5.00 bar. The piston is slowly moved to compress the gas to 8.00 bar.

(a) If the compression is carried out isothermally, and the work done on the gas equals 7.65 L-bar, how much heat (in joules) is transferred to or from (state which) the surroundings.

(b) Suppose instead that the process is adiabatic, what will happen to the temperature of the gas.

Example 15-1. Compression of an Ideal Gas in a Cylinder

(a) Isothermal Compression

1		2
4.00 L 30^0C 5.00 bar	Isothermal Compression → W = – 7.65 L-bar	V_2 30^0C 8.00 bar

This is a closed system and the energy balance is

$$\Delta K + \Delta P + \Delta U = Q - W$$

Since the system is stationary, $\Delta K = \Delta P = 0$
Since the process is isothermal, $\Delta T = 0$ and $\Delta U = 0$

The energy balance is simplified to

$$0 = Q - W$$

Solving for Q:

$$Q = W = -7.65\,\text{L} \cdot \text{bar}\left(\frac{8.314\,\text{J}}{0.08314\,\text{L} \cdot \text{bar}}\right) = -765\,\text{J}$$

(b) Adiabatic Compression

The energy balance reduces to $\Delta U = -W = -(-765\text{ J}) = 765\text{ J}$

Since ΔU is positive, ΔT must also be positive. Hence, the temperature of the gas will increase.

Example 15-2. Nitrogen Gas in a Piston-Fitted Cylinder

A piston-fitted cylinder with a 6-cm inner diameter contains 1.40 g of nitrogen. The mass of the piston is 4.50 kg, and a 20-kg weigh rests on the piston. The gas temperature is 30^0C and the pressure outside the cylinder is 1.00 atm.

(a) Calculate the pressure and volume of the gas inside the cylinder if the piston-weight is at equilibrium.

(b) Suppose the 20-kg weigh is abruptly lifted and the piston rises to a new equilibrium position in which the gas returns to 30^0C. Calculate the work done by the gas.

(a) Pressure of the gas inside the cylinder

If the piston-weight is at equilibrium, then

$$\mathbf{F_U = F_D}$$

Calculate F_D:

$$F_D = P_{ext}A_{piston} + W_{piston} + W_{20-kg}$$

The cross-sectional area of the piston is

$$A_{piston} = \pi r^2 = \pi(3\,cm)^2\left(\frac{1\,m}{100\,cm}\right)^2 = 2.83\times10^{-3}\,m^2$$

The force due to external pressure:

$$F_{ext} = P_{ext}A_{piston} = 1.00\,\text{atm}\left(\frac{101325\,\text{N}/\text{m}^2}{1.00\,\text{atm}}\right)\left(2.83\times10^{-3}\,\text{m}^2\right)$$

$$F_{ext} = 287\,\text{N}$$

The force due to piston and 20-kg mass:

$$F_P = (4.50\text{ kg} + 20.00\text{ kg})(9.81\text{ m/s}^2) = 240\text{ N}$$

The total downward force is

$$F_D = (240 + 287)\text{ N} = 527\text{ N} = F_U$$

Example 15-2. Nitrogen Gas in a Piston-Fitted Cylinder

$$P_{gas} = F_U/A_{piston} = 527\ N/(2.87 \times 10^{-3}\ m^2) = 1.86 \times 10^5\ N/m^2$$

Assuming the gas is ideal:

$$V = \frac{nRT}{P} = \frac{mRT}{(MW)(P)} = 0.677\,L$$

(b) Work done by the gas when 20-kg mass is removed

Upon removal of the 20-kg mass, the downward force is reduced to:

$$F_D = 287\ N + (4.50\ kg)(9.81\ m/s_2) = 331\ N$$

Example 15-2. Nitrogen Gas in a Piston-Fitted Cylinder

If the piston must attain a new equilibrium position, then

$$\mathbf{F_D = F_U = 331\ N}$$

The final pressure of the gas:

$$P_2 = F_U/A_{piston} = 331\ \text{N}/(2.87 \times 10^{-3}\ \text{m}^2) = 1.16 \times 10^5\ \text{N/m}^2$$

And the new volume of the gas is:

$$V_2 = V_1\left(\frac{P_1}{P_2}\right) = 0.677\,\text{L}\left(\frac{1.86\times10^5}{1.16\times10^5}\right) = 1.08\,\text{L}$$

Change in volume: $\Delta V = V_2 - V_1 = (1.08 - 0.677)\text{L} = 0.403\ \text{L}$

Example 15-2. Nitrogen Gas in a Piston-Fitted Cylinder

The change in volume can also be determined as:

$$\Delta V = A_{piston}\Delta x$$

Therefore, the displacement Dx can be computed as:

$$\Delta x = \Delta V/A_{piston} = 0.142 \text{ m}$$

Computing for work:

$$W = F\Delta x = (331 \text{ N})(0.142 \text{ m}) = +47 \text{ J}$$

Since $\Delta K = \Delta P = \Delta U = 0$, this work must be accompanied by heat transfer to the gas equal to +47 J.

Example 15-3. Methane Flowing in a Pipe

Methane enters a 3-cm ID pipe at 30^0C and 10 bar with an average velocity of 5.00 m/s and emerges at a point 200 m lower than the inlet at 30^0C and 9 bar.

Calculate the ΔK and ΔP assuming the methane behaves as an ideal gas.

Solution:

Mass flow must be the same at the inlet to attain steady-state condition.

$$\Delta K = m/_2(v_2^2 - v_1^2) \text{ and } \Delta P = mg(h_2 - h_1)$$

Determine the mass flow:

Volumetric flow at the inlet = v_1A

If methane behaves as an ideal gas:

$$V = v_1 A = \frac{mRT}{(MW)(P)}$$

Solving for mass flow:

$$m = \frac{(v_1 A)(MW)(P_1)}{RT_1} = 0.0225 \, kg/s$$

Solving for ΔP:

$$\Delta P = mg(h_2 - h_1) = \left(0.0225\frac{kg}{s}\right)\left(9.81\frac{m}{s^2}\right)(-200\,m)$$

$$\Delta P = -44.1\frac{J}{s} = -44\,W$$

Determine v_2:

$$P_1V_1 = P_2V_2$$

$$P_1(v_1A) = P_2(v_2A)$$

Solving for v_2:

$$v_2 = v_1\left(\frac{P_1}{P_2}\right) = 5.00\frac{m}{s}\left(\frac{10\,bar}{9\,bar}\right) = 5.555\,m/s$$

Solving for ΔK:

$$\Delta K = \frac{m}{2}\left(v_2{}^2 - v_1{}^2\right) = \frac{1}{2}\left(0.0225\frac{kg}{s}\right)\left(5.555^2 - 5.00^2\,\frac{m^2}{s^2}\right)$$

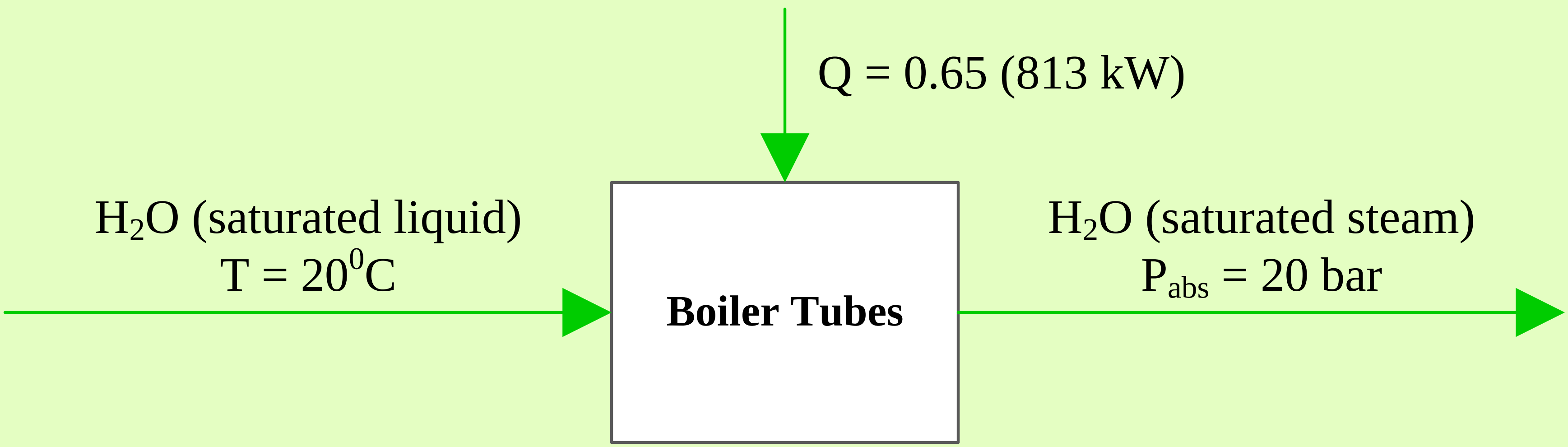
$$\Delta K = 0.0659\frac{J}{s} = 0.0659\,W$$

Example 15-4. Heating of Water in Boiler Tubes

A fuel oil is burned with air in a boiler furnace. The combustion produces 813 kW of heat of which 65% is transferred as heat to boiler that pass through the furnace. Water enters the boiler tubes as a saturated liquid at 20^0C and leaves the tubes as saturated steam at 20 bar absolute.

Calculate the mass flow rate (in kg/h) and volumetric flow rate (in m^3/h) at which the saturated steam is produced。

Example 15-4. Heating of Water in Boiler Tubes



Using the energy balance for an open system,

$$\Sigma(K + P + H)_2 - \Sigma(K + P + H)_1 = Q - W_S$$

Assuming $\Delta K = \Delta P = W = 0$, then

$$\Sigma H_2 - \Sigma H_1 = Q = 0.65\ (813\ kW) = 528\ kW$$

Example 15-4. Heating of Water in Boiler Tubes

In terms of specific enthalpy, $\mathbf{\hat{H}}$ (kJ/kg)

$$\mathbf{m_2\hat{H}_2} - \mathbf{m_1\hat{H}_1} = 528 \text{ kW}$$

where m = mass flow rate of water

If the process is under steady-state condition, then $\mathbf{m_1 = m_2 = m}$

Hence,

$$\mathbf{m(\hat{H}_2} - \mathbf{\hat{H}_1)} = 528 \text{ kW}$$

Solving for m:

$$m = \frac{528 \text{kW}}{\hat{H}_2 - \hat{H}_1}$$

Example 15-4. Heating of Water in Boiler Tubes

From saturated steam table,

$$\mathbf{\hat{H}_1} = 83.9 \text{ kJ/kg and } \mathbf{\hat{H}_2} = 2797.2 \text{ kJ/kg}$$

And the mass flow rate is:

$$m = \frac{528 \text{ kJ/s}}{(2797.2 - 83.9) \text{ kJ/kg}} \left(\frac{3600 \text{ s}}{1 \text{ h}} \right) = 701 \text{ kg/h}$$

Calculating for the volume flow rate:

$$V = m\hat{V} = 701 \frac{\text{kg}}{\text{h}} \left(0.0995 \frac{\text{m}^3}{\text{kg}} \right) = 69.7 \frac{\text{m}^3}{\text{h}}$$

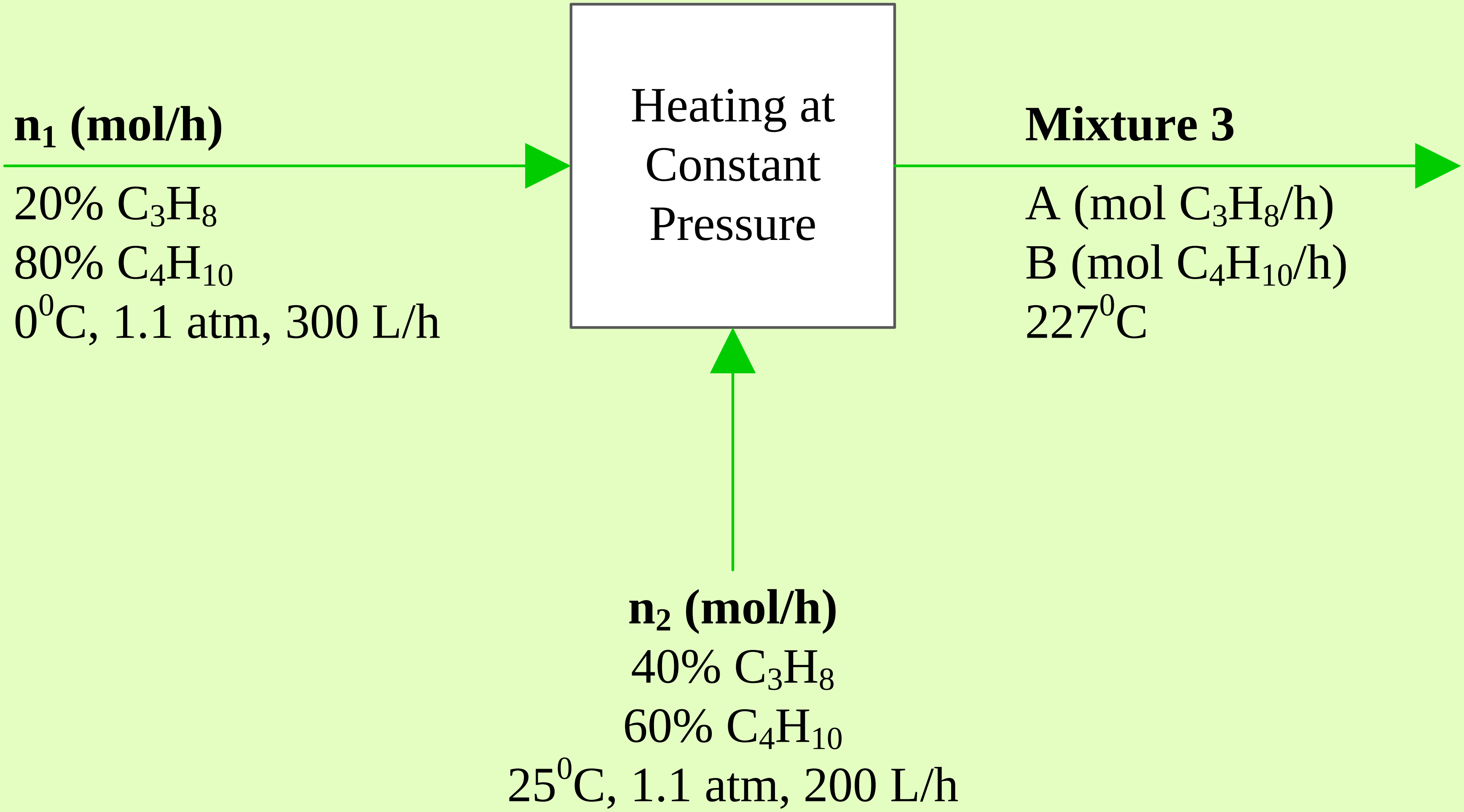
↑ from steam table

Example 15-5. Mixing and Heating of Propane-Butane Mixtures

Three hundred L/h of 20 mole% C_3H_8-80% n-C_4H_{10} gas mixture at 0^0C and 1.1 atm and 200 L/h of a 40 mole% C_3H_8-60% n-C_4H_{10} gas mixture at 25^0C and 1.1 atm are mixed and heated to 227^0C at constant pressure. Calculate the heat requirement of the process. Enthalpies of propane and n-butane are listed below. Assume ideal gas behaviour.

T (^{0}C)	**Propane** $\hat{H}$ (J/mol)	**Butane** $\hat{H}$ (J/mol)
0	0	0
25	1772	2394
227	20,685	27,442

Example 15-5. Mixing and Heating of Propane-Butane Mixtures



Simplified energy balance for open system:

$$Q = \Sigma H_{out} - \Sigma H_{in}$$
$$Q = (H_{P3} + H_{B3}) - (H_{P2} + H_{B2} + H_{P1} + H_{B1})$$

Since mixture 1 is at 0^0C, then

$$Q = (H_{P3} + H_{B3}) - (H_{P2} + H_{B2})$$

The total enthalpy of each component is determined as:

$$H_{P3} = A\hat{H}_{P3} \qquad H_{B3} = B\hat{H}_{B3}$$
$$H_{P2} = 0.40n_2\hat{H}_{P2} \qquad H_{B2} = 0.60n_2\hat{H}_{B3}$$

Find n_1, n_2, A, and B

n_1 and n_2 can be obtained from V_1 and V_2 using the ideal gas equation:

$$n_1 = 14.7 \text{ mol/h} \; ; \; n_2 = 9.00 \text{ mol/h}$$

A and B can be obtained using material balances for propane and butane:

Propane: $A = 0.20n_1 + 0.40n_2 = 6.54 \text{ mol } C_3H_8\text{/h}$

Butane: $B = 0.80n_1 + 0.60n_2 = 17.16 \text{ mol } C_4H_{10}\text{/h}$

Solving for the total enthalpies of the components:

$$H_{P3} = (6.54 \text{ mol/h})(20.865 \text{ kJ/mol}) = 136.5 \text{ kJ/h}$$
$$H_{B3} = (17.16 \text{ mol/h})(27.442 \text{ kJ/mol}) = 470.9 \text{ kJ/h}$$
$$H_{P2} = 0.40(9.00 \text{ mol/h})(1.772 \text{ kJ/mol}) = 6.38 \text{ kJ/h}$$
$$H_{B2} = 0.60(9.00 \text{ mol/h})(2.394 \text{ kJ/mol}) = 12.93 \text{ kJ/h}$$

Solving for the heat requirement of the process:

$$Q = (136.5 + 470.9 - 6.38 - 12.93) \text{ kJ/h}$$
$$\mathbf{Q = 587 \text{ kJ/h}}$$