CHE 31. Introduction to Chemical Engineering

**Problem Set 3. Material Balances**

(Problems adopted from Elementary Principles of Chemical Process by Felder & Rousseau and Basic Principles and Calculations in Chemical Engineering by Himmelblau and Briggs)

1. Air is bubbled through a drum of liquid hexane at a rate of 0.100 kmol/min. The gas stream leaving the drum contains 10.0 mole% hexane vapor. Air maybe considered insoluble in liquid hexane. Use an integral balance to estimate the time required to vaporize 10.0 $m^3$ of the liquid. **Ans. t = 6880 min**

2. One thousand kilograms per hour of a mixture containing equal parts by mass of methanol and water is distilled. Product streams leave the top and the bottom of the distillation column. The flow rate of the bottom stream is measured and found to be 673 kg/h, and the overhead stream is analyzed and found to contain 96.0 wt% methanol. Calculate the mass and mole fractions of methanol and the molar flow rates of methanol and water in the bottom product stream.

   **Ans.** $\mathbf{X_M = 0.276}$ **kg** $\mathbf{CH_3OH/kg}$ $\mathbf{Y_M = 0.176}$ **mol** $\mathbf{CH_3OH/mol}$
   $\mathbf{m_M = 5.80 \times 10^3}$ **mol** $\mathbf{CH_3OH/h}$ $\mathbf{m_W = 2.71 \times 10^4}$ **mol** $\mathbf{H_2O/h}$

3. A paint mixture containing 25.0 wt% of a pigment and the balance water sells for $18.00/kg, and a mixture containing 12.0 wt% pigment sells for $10.00/kg. If a paint retailer produces a blend containing 17.0 wt% pigment, for how much ($/kg) should it be sold to yield a 10% profit? **Ans. P = $ 14.39 / kg**

4. An artificial kidney is a device that removes water and waste metabolites from blood. In one such device, the hollow fiber hemodialyzer, blood flows from an artery through the insides of a bundle of hollow cellulose acetate fibers, and dialyzing fluid, which consists of water and various dissolved salts, flows on the outside of the fibers. Water and waste metabolites-principally urea, creatinine, uric acid, and phosphate ions-pass through the fiber walls into the dialyzing fluid, and the purified blood is returned to a vein.

   At some time during dialysis the arterial and venous blood conditions are as follows:

| | Arterial Blood (Entering the Device) | Venous Blood (Exiting the Device) |
|---|---|---|
| Flow Rate | 200.0 mL/min | 195.0 mL/min |
| Urea ($H_2NCONH_2$) Concentration | 1.90 mg/mL | 1.75 mg/mL |

   a. Calculate the rates at which urea and water are being removed from the blood.
      **Ans. Water=5.0 mL/min, Urea=38.8 mg/min**

b. If the dialyzing fluid enters at a rate of 1500 mL/min and the exiting solution (dialysate) leaves at approximately the same rate, calculate the concentration of urea in the dialysate. **Ans. 0.0258 mg urea/mL**

c. Suppose we want to reduce the patient's urea level from an initial value of 2.7 mg/mL to a final value of 1.1 mg/mL. If the total blood volume is 5.0 liters and the average rate of urea removal is that calculated in part (a), how long must the patient be dialyzed? (Neglect the loss in total blood volume due to removal of water in the dialyzer. **Ans. 206 min**

5. Seawater containing 3.50 wt% salt passes through a series of 10 evaporators. Roughly equal quantities of water are vaporized in each of the 10 units and hen condensed and combined to obtain a product stream of fresh water. The brine leaving each evaporator but the tenth is fed to the next evaporator. The brine leaving the tenth evaporator contains 5.00 wt% salt. Determine the fractional yield of fresh water from the process (kg $H_2O$ recovered/kg $H_2O$ in process feed) and the fraction of salt in the solution leaving the fourth evaporator.

   **Ans. $Y_W = 0.31$ ; $x_4 = 0.0398$**

6. In an absorption tower (or absorber), a gas is contacted with a liquid such that one or more components in the gas is transferred in the liquid. A stripping tower (stripper) also involves a gas contacting a liquid but components are transferred from the liquid into the gas.

   A process consisting of an absorber and a stripper is used to separate the components of a gas containing 30.0 mole% $CO_2$ and the balance $CH_4$. This gas is fed to the absorber. A liquid containing 0.500 mole% $CO_2$ and the balanced methanol is recycled from the stripper and also fed to the absorber.

   The effluent gas leaving the absorber contains 1.00 mole% $CO_2$ and all the $CH_4$ in the feed gas. The effluent solvent leaving the absorber is fed to the stripper (the same stripper that supplies the recycled $CO_2$-methanol liquid mixture) together with a stream of nitrogen gas. Ninety percent of the $CO_2$ in this effluent solvent is removed in the stripper and the nitrogen/$CO_2$ stream leaving the stripper passes out to the atmosphere. Methanol may be assumed to be nonvolatile and nitrogen may be assumed insoluble in methanol.

   Taking 100 mol/h of the gas fed to the absorber, calculate the

   a. Fractional $CO_2$ removal in the absorber (moles $CO_2$ absorbed/mole $CO_2$ in gas feed). **Ans. 0.976 mol $CO_2$ absorbed/mol fed**

   b. Molar flow rate and composition of the liquid feed to the stripping tower. **Ans. 680 mol/h; 0.0478 mol $CO_2$/mol**

   c. Molar feed rate of the gas to the absorber required to produce an absorber product gas flow rate of 1000 kg/h. **Ans. 8.69 x $10^4$ mol/h**

7. Fresh orange juice contains 12.0 wt% solids and the balance water, and the concentrated orange juice contains 42.0 wt% solids. Initially a single evaporation process was used for the concentration, but volatile constituents of the juice escaped with the water, leaving the concentrate with a flat taste. The current process overcomes this problem by bypassing the evaporator with a fraction of the fresh juice. The juice is concentrated to 58 wt% solids, and the evaporator product stream is mixed with the bypassed fresh juice to achieve the desired final concentration. Calculate the amount of product (42% concentrate) produced per 100 kg fresh juice fed to the process and the fraction of the feed that bypasses the evaporator.

   **Ans. 28.6 kg product per 100 kg feed; bypass fraction = 0.095**

8. A stream containing 5.15 wt% chromium, Cr, is contained in the wastewater from a metal finishing plant. The wastewater stream is fed to a treatment unit that removes 95% of the chromium in the feed and recycles it to the plant. The residual liquid stream leaving the treatment unit is sent to a waste lagoon. The treatment unit has a maximum capacity of 4500 kg wastewater/h. If wastewater leaves the finishing plant at a rate higher than the capacity of the treatment unit, the excess (anything above 4500 kg/h) bypasses the unit and combines with the liquid leaving the unit, and the combined streams goes to the waste lagoon. If wastewater leaves the finishing plant at a rate of 6000 kg/h, calculate the flow rate of liquid to the waste lagoon (kg/h) and the mass fraction of Cr in the liquid (kg Cr/kg).

   **Ans. 5779.8 kg/h, 0.0154 kg Cr/kg**

9. In the production of bean oil, beans containing 13.0 wt% oil and 87.0 wt% solids are ground and fed to an extractor along with fresh and recycled streams of liquid n-hexane. The feed ratio is 3kg hexane/kg beans. During extraction, the ground beans are suspended in the liquid, and essentially all of the oil in the beans is extracted into the hexane.

   The extractor effluent then passes to a filter. The filter cake contains 75.0 wt% bean solids and the balance bean oil and hexane, the latter two in the same ratio in which they emerge from the extractor. The filter cake is discarded and the liquid filtrate is fed to a heated evaporator in which the hexane is vaporized and the oil remains as a liquid. The oil is then taken as the process product. The hexane vapor is subsequently cooled and condensed, and the liquid condensate is recycled to the extractor.

   Calculate the yield of bean oil product (kg oil/kg beans fed), the required fresh hexane feed per kilogram of beans fed, and the recycle to fresh hexane feed ratio.

   **Ans. Yield=0.118 oil/kg beans fed**
   **Fresh hexane feed=0.28 kg $C_6H_{14}$/kg beans fed**
   **Recycle ratio=9.71 kg recycled $C_6H_{14}$/kg fresh $C_6H_{14}$**

10. Titanium dioxide ($TiO_2$) is used extensively as a white pigment. It is produced from an ore that contains ilmenite ($FeTiO_3$) and ferric oxide ($Fe_2O_3$). The ore is digested with an aqueous

sulfuric acid solution to produce an aqeous solution of titanyl sulfate [$(TiO)SO_4$] and ferrous sulfate ($FeSO_4$). Water is added to hydrolyze the titanyl sulfate to $H_2TiO_3$, which precipitates, and $H_2SO_4$. The precipitate is then roasted, driving off water and leaving a residue of pure titanium dioxide.

Suppose an ore containing 24.3% Ti by mass is digested with 80% $H_2SO_4$ solution, supplied in 50% excess of the amount needed to convert all the ilmenite to titanyl sulfate and all the ferric oxide to ferric sulfate [$Fe_2(SO_4)_3$]. Further, suppose that 89% of the ilmenite actually decomposes. Calculate the masses (kg) of ore and 80% sulfuric acid solution that must be fed to produce 1000 kg of pure $TiO_2$.

**Ans. 2772 kg ore, 7380 kg 80% $H_2SO_4$**

11. Coal containing 5.0 wt% S is burned at a rate of 1250 $lb_m$/min in a boiler furnace. All of the sulfur in the coal is oxidized to $SO_2$. The product gas is sent to a scrubber in which most of the $SO_2$ is removed, and the scrubbed gas then passes out of a stack. An Environmental Protection Agency (EPA) regulation requires that the gas in the stack must contain no more than 0.018 $lb_m$ SO2/$lb_m$ coal burned. To test compliance with this regulation a flowmeter and an $SO_2$ analyzer are mounted in the stack. The volumetric flow rate of the scrubbed gas is found to be 2867 $ft^3$/s, and the $SO_2$ analyzer reading is 37. Calibration data for the analyzer are given in the table below.

$SO_2$ Analyzer Calibration Data

| $C$ (g $SO_2$/$m^3$ gas) | Reading (0-100 scale) |
|---|---|
| 0.30 | 10 |
| 0.85 | 28 |
| 2.67 | 48 |
| 7.31 | 65 |
| 18.20 | 81 |
| 30.00 | 90 |

a. Determine the equation that relates $SO_2$ concentration in $lb_m/ft^3$ to analyzer reading.
**Ans. $C'(lb_m\ SO_2/ft^3) = 1.055\text{x}10^{-5}\ e^{0.0575R}$**

b. Determine the lbm $SO_2$/lbm coal burned in the process to check whether it complies with the EPA regulation. **Ans. 0.012 $lb_m$ $SO_2$/$lb_m$ coal**

c. What percentage of the $SO_2$ produced in the furnace is removed in the scrubber?
**Ans. 88%**

12. Ethanol is produced commercially by the hydration of ethylene:

$$C_2H_4 + H_2O \rightarrow C_2H_5OH$$

Some of the product is converted to diethyl ether in the side reaction

$$2\ C_2H_5OH \rightarrow (C_2H_5)_2O + H_2O$$

The feed to the reactor contains ethylene, steam, and an inert gas. A sample of the reactor effluent gas is analyzed and found to contain 43.3 mole% ethylene, 2.5% ethanol, 0.14% diethyl ether, 9.3% inerts, and the balance water. Calculate (a) the molar composition of the reactor feed; (b) the percentage conversion of ethylene; (c) the fractional yield of ethanol and (d) the selectivity of ethanol production relative to diethyl ether production.

**Ans.**
**(a) Reactor feed contains 44.8% $C_2H_4$, 46.1% $H_2O$, 9.1%I**
**(b) % Conversion of $C_2H_4$ = 6.0%**
**(c) Fractional yield of ethanol = 0.054**
**(d) Selectivity = 17.9 mol $C_2H_5OH$/mol $(C_2H_5)_2O$**

13. Solid calcium fluoride ($CaF_2$) reacts with sulfuric acid to form calcium sulfate and gaseous hydrogen fluoride (HF). The HF is then dissolved in water to form hydrofluoric acid. A source of calcium fluoride is fluorite ore containing 96.0 wt% $CaF_2$ and 4.0% $SiO_2$.

    In a typical hydrofluoric acid manufacturing process, fluorite ore is reacted with 93 wt% aqueous sulfuric acid, supplied 15% in excess of the stoichiometric amount. Ninety-five percent of the ore dissolves in the acid. Some of the HF formed reacts with the dissolved silica in the reaction

    $$6HF + SiO_{2\,(aq)} \rightarrow H_2SiF_{6\,(s)} + 2\ H_2O_{\,(l)}$$

    The hydrogen fluoride exiting from the reactor is subsequently dissolved in enough water to produce 60.0 wt% hydrofluoric acid. Calculate the quantity of fluorite ore needed to produce a metric ton of acid. **Ans. 1533 kg ore**

14. Chlorobenzene ($C_6H_5Cl$), an important solvent and intermediate in the production of many other chemicals, is produced by bubbling chlorine gas through liquid benzene in the presence of ferric chloride catalyst. In an undesired side reaction, the product is further chlorinated to dichlorobenzene, and in a third reaction, the dichlorobenzene is chlorinated to trichlorobenzene.

    The feed to a chlorination reactor consists of essentially pure benzene and a technical grade of chlorine gas (98 wt% $Cl_2$, the balance gaseous impurities with an average molecular weight of 25.0). The liquid output from the reactor contains 65.0 wt% $C_6H_6$, 32.0 wt% $C_6H_5Cl$, 2.5 wt% $C_6H_4Cl_2$, and 0.5 wt% $C_6H_3Cl_3$. The gaseous output contains only HCl and the impurities that entered with the chlorine.

    Determine (a) the percentage by which benzene is fed in excess, (b) the fractional conversion of benzene, (c) the fractional yield of monochlorobenzene, and (d) the mass ratio of the gas feed to the liquid feed.

**Ans.**
**(a) 246% excess $C_6H_6$**
**(b) Fractional Conversion of $C_6H_6$ = 0.268 mol $C_6H_6$ react/mol fed,**
**(c) Yield = 0.865**
**(d) Mass Ratio = 0.268 g gas/g liquid**

15. A catalytic reactor is used to produce formaldehyde from methanol in the reaction:

$$CH_3OH \rightarrow HCHO + H_2$$

A single-pass conversion of 60.0% is achieved in the reactor. The methanol in the reactor product is separated from the formaldehyde and hydrogen in a multiple-unit process. The production rate of formaldehyde is 900 kg/h.

a. Calculate the required feed rate of methanol to the process (kmol/h) if there is no recycle. **Ans. 50.0 kmol $CH_3OH$/h**

b. Suppose the recovered methanol is recycled to the reactor and the single-pass conversion remains 60%, determine the required fresh feed rate of methanol (kmol/h) and the rates (kmol/h) at which methanol enters and leaves the reactor.

**Ans. Fresh Feed = 30.0 kmol $CH_3OH$/h, Methanol Rate = 20.0 kmol $CH_3OH$/h**

16. Ethylene oxide is produced by the catalytic oxidation of ethylene:

$$2C_2H_4 + O_2 \rightarrow 2C_2H_4O$$

An undesired competing reaction is the combustion of ethylene:

$$C_2H_4 + 3O_2 \rightarrow 2CO_2 + 2H_2O$$

The feed to the reactor (not the fresh feed to the process) contains 3 moles of ethylene per mole of oxygen. The single-pass conversion of ethylene is 20%, and for every 100 moles of ethylene consumed in the reactor, 90 moles of ethylene oxide emerges in the reactor products. A multiple-unit process is used to separate the products: ethylene and oxygen are recycled to the reactor, ethylene oxide is sold as a product, and carbon dioxide and water are discarded. Calculate (a) the molar flow rates of ethylene and oxygen in the fresh feed, (b) the production rate of ethylene oxide, (c) the overall conversion of ethylene, and (d) the molar flow rates of ethylene and oxygen in the fresh feed needed to produce 1 ton per hour of ethylene oxide.

**Ans. (a) 11.25 mol $O_2$/h, 15.0 mol/h $C_2H_4$; (b) 13.5 mol $C_2H_4O$/h;**
**(c) Overall Conversion = 100%; (d) 50.4 lbmol $C_2H_4$/h, 37.8 lbmol $O_2$/h**

17. Five liters of liquid n-hexane and 4 liters of liquid n-heptane are mixed and burned with 400 mol of air. Not all of the hydrocarbons are burned in the furnace, and both CO and CO2 are formed. Calculate the percent excess air supplied to the furnace. **Ans. 26.3% excess air**

18. A mixture of 75 mole% propane and 25 mole% hydrogen is burned with 25% excess air. Fractional conversions of 90% of the propane and 85% of the hydrogen are achieved; of the propane that reacts, 95% reacts to form CO2 and the balance reacts to form CO. The hot combustion product gas passes through a boiler in which heat transferred from the gas converts boiler feedwater into steam. Calculate the concentration of CO (ppm) in the stack gas.

    **Ans. Concentration of CO in exit gas = 4090 ppm**

19. n-Pentane is burned with excess air in a continuous combustion chamber. A technician runs an analysis and reports that the product gas contains0.304 mole% pentane, 5.9% oxygen, 10.2% carbon dioxide, and the balance nitrogen on a dry basis. Calculate the percent excess air fed to the reactor and the fractional conversion of pentane.

    **Ans. 18.6% excess air, Fractional Conversion of Pentane = 0.870**

20. A fuel oil is analyzed and found to contain 85.0 wt% carbon, 12.0% elemental hydrogen (H), 1.7% sulfur, and the remainder noncombustible matter. The oil is burned with 20.0% excess air, based on complete combustion of the carbon to $CO_2$, the hydrogen to $H_2O$, and the sulfur to $SO_2$. The oil is burned completely, but 8% of the carbon forms CO. Calculate the molar composition of the stack gas.

    **Ans. 10.7% $CO_2$, 0.92% CO, 0.087% $SO_2$, 9.8% $H_2O$, 3.8% $O_2$, 74.8% $N_2$**

21. The analysis of coal indicates 75 wt% C, 17% H, 2% S, and the balance noncombustible ash. The coal is burned at a rate of 5000 kg/h, and the feed rate of air to the furnace is 50 kmol/min. All of the ash and 6% of the carbon in the fuel leave the furnace as a molten slag; the remainder of the carbon leaves in the stack gas as CO and $CO_2$; the hydrogen in the coal is oxidized to water; and the sulfur emerges as $SO_2$. The selectivity of $CO_2$ to CO production is 10:1.

    a. Calculate the percent excess air fed to the reactor. **Ans. 19.8% excess air**

    b. Calculate the mole fractions of the gaseous pollutants – CO and $SO_2$ – in the stack gas. **Ans. 8.3 x $10^{-3}$ mol CO/mol, 9.6 x $10^{-4}$ mol SO2/mol**

    c. Emitted sulfur dioxide by itself is a health hazard, but it is a more serious threat to the environment as a precursor to acid rain. Under the catalytic action of sunlight, the $SO_2$ is oxidized to $SO_3$, which in turn combines with water vapor to form sulfuric acid, which eventually returns to earth as rainfall. Acid rain formed in this manner has caused extensive damage to forests, fields, and lakes in many parts of the world. For the furnace described above, calculate the rate of formation of sulfuric acid (kg/h) if all the emitted $SO_2$ is converted in the indicated manner.

       **Ans. 304 kg $H_2SO_4$/h**

22. Plants in Europe sometimes use the mineral pyrites (the desired compound in the pyrites is ($FeS_2$) as a source of $SO_2$ for the production of sulfite pulping liquor. Pyrite rock containing 48.0 % sulfur is burned completely by flash combustion.

    All of the iron forms $Fe_3O_4$ in the cinder (the solid product), and a negligible amount of $SO_3$ occurs in either the cinder or the product gas. The gas from such a furnace is passed through milk of lime (CaO in water) absorbers to produce bisulfite pulping liquor. The exit gas from the absorber analyzes: $SO_2$ 0.7 %, $O_2$ 2.9 % and $N_2$ 96.4 %.

    Calculate the kg of air supplied to the burner per kg of the pyrites burned.

    **Ans. 3.1 kg air/ kg pyrites**